THE STRUCTURE OF A MAGNETOHYDRODYNAMIC SHOCK WAVE IN A GAS WITH AN ANISOTROPIC CONDUCTIVITY

(O STRUKTURE MAGNITOGIDRODINAMICHESKOI UDARNOI VOLNY V GAZE S ANIZOTROPNOI PROVODIMOST'IU)

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It is demonstrated in [1] that the thickness of a shock wave in a non-ideal medium may not tend toward zero when all dissipative coefficients tend toward zero. Below we will construct an example of such a shock wave.

Let us examine a problem dealing with the structure of a magnetohydrodynamic shock wave in a gas with an anisotropic conductivity in terms of the situation described in [2], where the following form of a generalized Ohm's law is employed:

$$c\mathbf{E} + \mathbf{v} \times \mathbf{H} + \frac{c}{ne} \operatorname{grad} p_e = \frac{c}{\sigma}\mathbf{j} + \frac{c}{\sigma}\frac{\omega\tau}{H}\mathbf{j} \times \mathbf{H}$$

Equations describing one-dimensional stationary motion may be reduced here to the form [2-4]:

$$\nu_m^* \frac{dH_y}{dx} + \nu_m^* \varkappa \frac{dH_z}{dx} = \frac{\partial P}{\partial H_y}, \qquad \nu_m^* = \frac{\nu_m}{4\pi T} = \frac{c^2}{16\pi^2 \sigma T}, \quad \varkappa = \frac{H_x}{H}\omega\tau$$

$$-\nu_m^* \varkappa \frac{dH_y}{dx} + \nu_m^* \frac{dH_z}{dx} = \frac{\partial P}{\partial H_z}, \qquad P = \frac{m}{T}\left[\frac{H_y^2 V}{8\pi} + \frac{H_z^2 V}{8\pi} + \frac{m^2 V^2}{2} + \frac{v^2}{2} + \frac{w^2}{2} - \right.$$

$$\left. - f(V,T) + E_0 H_y - H_0 H_y v - H_0 H_z w - JV + S\right] \qquad \left(m = u\rho, \ V = \frac{1}{\rho}\right)$$

In the above formulas u, v and w are velocity components, $f(V, T)$ the free energy and E_0, H_0, J, and S constants. In expressions $\partial P/\partial H_y$ and $\partial P/\partial H_z$ all values must be expressed in terms of H_y and H_z from the relations

$$\frac{\partial P}{\partial V} = 0, \qquad \frac{\partial P}{\partial v} = 0, \qquad \frac{\partial P}{\partial w} = 0, \qquad \frac{\partial P}{\partial T} = 0$$

The matrix of the dissipative coefficients designated in [1] by $\|L_{ij}\|$, and its reverse matrix $\|\Lambda_{ij}\|$ has, in the present instance, the form:

$$L_{ij} \equiv \begin{Vmatrix} \nu_m^* & \varkappa\nu_m^* \\ -\varkappa\nu_m^* & \nu_m^* \end{Vmatrix}, \qquad \Lambda_{ij} = \frac{1}{\nu_m^{*2}(1+\varkappa^2)} \begin{Vmatrix} \nu_m^* & -\varkappa\nu_m^* \\ \varkappa\nu_m^* & \nu_m^* \end{Vmatrix}$$

The thickness of the shock wave can not be less than the value

$$l = \int \frac{dP}{\Lambda_{ij}\ (\partial P/\partial H_i)(\partial P/\partial H_j)} = \nu_m^*(1+\varkappa^2)\int \frac{dP}{(\partial P/\partial H_y)^2 + (\partial P/\partial H_z)^2} \qquad (i, j = y, z)$$

where integration with respect to P is carried out from the limit $P(S_1) + \varepsilon$ to $P(S_2) - \varepsilon$ along an integral curve connecting the singular points S_1 and S_2.

Within the limits of integration, $(\partial P/\partial H_y)^2 + (\partial P/\partial H_z)^2 > \varepsilon_1 (\varepsilon_1 > 0)$, which depends on ε. Hence

$$l \sim \nu_m^*(1+\varkappa^2)\,U^{-1}$$

where U is a certain characteristic of velocity.

This expression shows that when the dissipative coefficients ν_m^* and $\kappa\nu_m^*$ tend toward zero, the thickness of the shock wave behaves in the following manner:

$$l \to 0, \quad \text{if} \quad \nu_m^*\varkappa^2 \to 0; \quad l \nrightarrow 0, \quad \text{if} \quad \nu_m^*\varkappa^2 \nrightarrow 0$$

The last case is observed only when $\omega\tau \to \infty$. With great values of $\omega\tau$, problems dealing with the structure of the shock wave have a periodic character [2].

Since with great values of $\omega\tau$ the derivatives $\partial H_y/dx$, dH_z/dx are determined by terms which are not located on the main diagonal of matrix L_{ij} and which have the order

$$\frac{\varkappa}{\nu_m^*(1+\varkappa^2)U} \sim \frac{1}{\nu_m^*\varkappa U} \qquad (U \text{ is the characteristic of velocity})$$

the thickness of a period turns out to be of the order of $U\kappa\nu_m^*$ and tends toward zero, if the dissipative coefficients tend toward zero. The evaluation of the thickness of the shock wave given in [2] is justified for small values of $\omega\tau$, at large values of $\omega\tau$ it corresponds to the thickness of one period.

It is known from [4] that P represents the entropy flow and that the rate of increase of the entropy is given by the formula

$$\frac{dP}{dx} = L_{ij}\frac{dH_i}{dx}\frac{dH_j}{dx} = \nu_m{}^*\left[\left(\frac{dH_y}{dx}\right)^2 + \left(\frac{dH_z}{dx}\right)^2\right] \qquad (i,\ j = y,\ z)$$

If $\omega\tau = 0$, i.e. if L_{ij} is a diagonal matrix, the thickness of the shock wave tends toward zero (according to [3]), when $\nu_m{}^* \to 0$.

Nondiagonal elements of the matrix L_{ij} do not contribute to the expression dP/dx. If $\nu_m{}^* \to 0$ and $\nu_m{}^*\kappa \to 0$, but $\nu_m{}^*\kappa^2$ does not tend toward zero, then those terms prevent too rapid an increase in the derivatives dH_y/dx, dH_z/dx, in the above limiting process thus assuring the finiteness of the derivative dP/dx.

When $\nu_m{}^* \to 0$ but the value $\nu_m{}^*\kappa$, equal to $cH_x/16\sigma neT$, remains finite, $dP/dx \to 0$ and $l \to \infty$. At any finite interval $[x_1, x_2]$ here the solution tends toward a periodic solution with a finite period, and the entropy in this interval does not increase. Such a solution may, to all appearances, be regarded as a macroscopic analogue of the corresponding solution for plasma without dissipation.

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